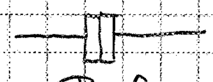
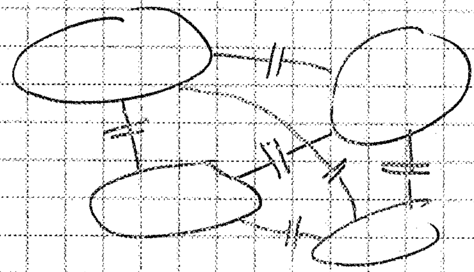
single charge devices

tunnel junction



R_t, C



$Q = CU$

for several islands $\vec{Q} = \overleftrightarrow{C}\,\vec{U}$

$$E = \int I U \, d\tau = \int \frac{dQ}{dt} U = \frac{1}{2} C U^2 = \frac{Q^2}{2C}$$

charging energy $E_c = \frac{e^2}{2C}$

example: standard electronics capacitor

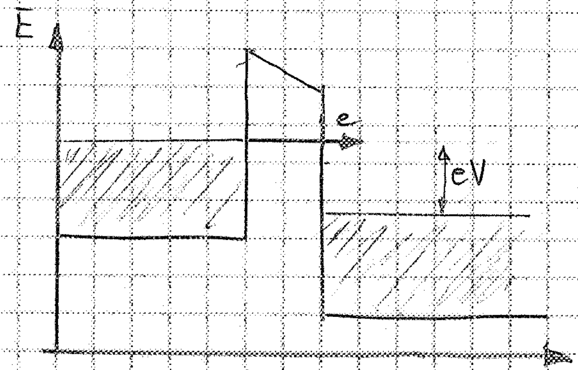
$C = 100\,\text{nF} \Rightarrow E_c = 1.3 \cdot 10^{-31}\,\text{J} = 9\,\text{nK} \cdot k_B$

parallel plate capacitor $C = \varepsilon \varepsilon_0 A/d$

example: shadow evaporated $Al - Al_2O_3 - Al$ junction

$A = (100\,\text{nm})^2 \qquad d = 1\,\text{nm}$

$\Rightarrow C = 9 \cdot 10^{-17}\,\text{F} \qquad E_c = 1.4 \cdot 10^{-22}\,\text{J} = 10\,\text{K} \cdot k_B$



tunneling rate (derived from Fermi's golden rule)

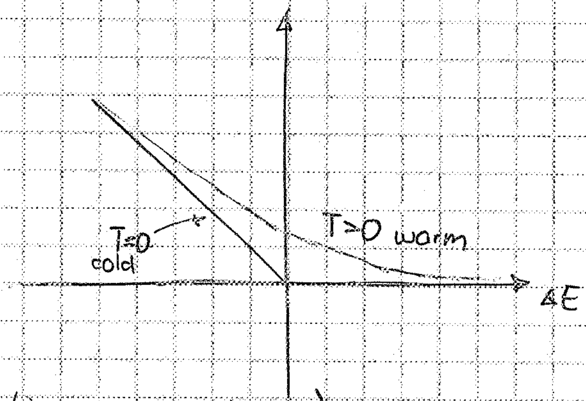
$$\Gamma = \frac{1}{e^2 R_t} \int d\varepsilon_i \int d\varepsilon_f \; f(\varepsilon_i)\left(1 - f(\varepsilon_f)\right) \delta(\varepsilon_i - \varepsilon_f - \Delta E)$$

empty final states

filled initial states

with $f(\varepsilon) = \frac{1}{e^{\varepsilon} - 1}$

$$\Gamma = \frac{1}{e^2 R_t} \frac{\Delta E}{e^{\frac{\Delta E}{k_B T}} - 1}$$



for low temperatures (i.e. $\Delta E \gg k_B T$)
tunneling is only possible if $\Delta E < 0$

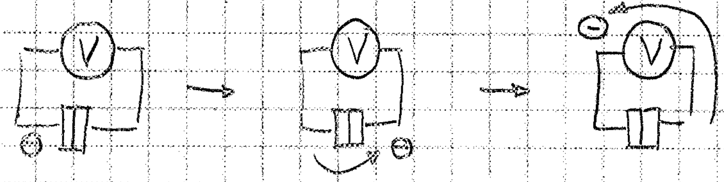
tunneling through a single junction with an ideal voltage source

$$\Delta E = E_f - E_i = \frac{(Q-e)^2}{2C} - \frac{Q^2}{2C} = -\underbrace{\frac{2Qe}{2C}}_{=-Ve} + \frac{e^2}{2C}$$

tunneling possible if $\Delta E < 0 \Rightarrow V > \frac{e}{2C}$

if $U < \frac{e}{2C}$ and $E_c \gg k_B T \Rightarrow$ Coulomb blockade

condition on the junction resistance



time to charge the leads $\Delta t = R_t C$

Heisenberg uncertainty relation

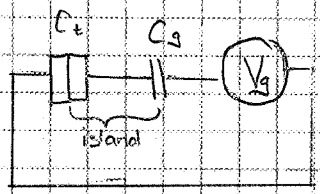
$$\frac{\hbar}{2} < \Delta E \, \Delta t = \frac{e^2}{2C} R_t C$$

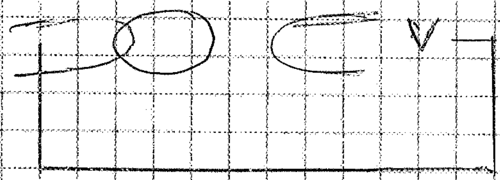
$$R_t > \frac{h}{e^2} = R_q = 25\,k\Omega$$

resistance quantum
von Klitzing constant

Coulomb blockade observed, if $R_t > R_q$

single electron box





$$Q_i = Q_t + Q_g$$

$$\frac{Q_t}{C_t} + \frac{Q_g}{C_g} = V_g$$

$$Q_g = \frac{C_g}{C_t + C_g}\left(C_t V_g - Q_i\right)$$

$$Q_t = \frac{C_t}{C_t + C_g}\left(C_g V_g + Q_i\right)$$

electrostatic energy

$$E\left(V_g, Q_i\right) = \frac{Q_t^2}{2C_t} + \frac{Q_g^2}{2C_g} = \frac{C_t C_g V_g^2 + Q_i^2}{2\, C_\Sigma}$$

$$C_\Sigma = C_t + C_g$$

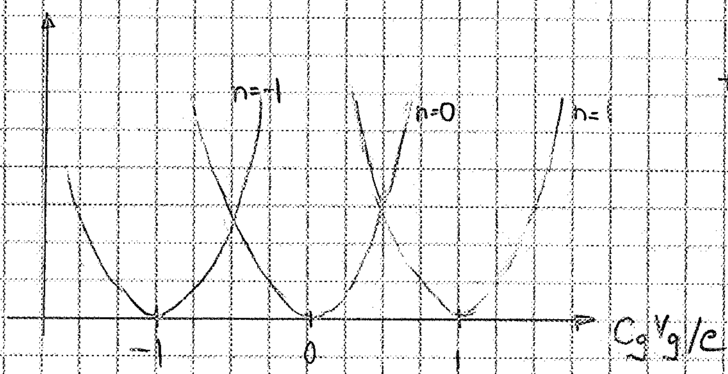
since the voltage source does work
Gibbs' free energy

$$G\left(V_g, Q_i\right) = E - Q_g V_g$$

$$= \frac{\left(C_g V_g + Q_i\right)^2}{2\, C_\Sigma} - \frac{C_g V_g^2}{2}$$

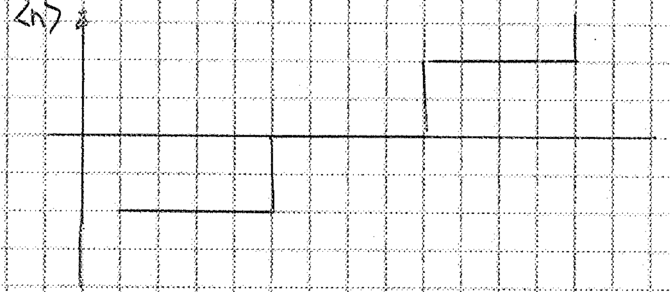
$Q_i = -ne \qquad n = \ldots, -2, -1, 0, 1, \ldots$

$$G(V_g, n) = E_c\left(n - C_g V_g/e\right)^2 \qquad E_c = \frac{e^2}{2C_\Sigma}$$

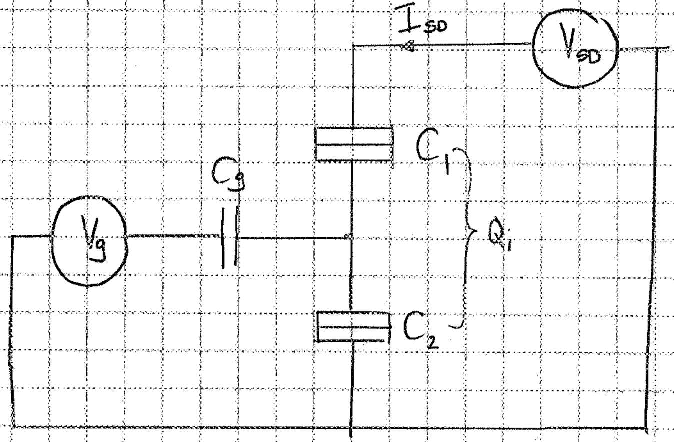


tunneling, if

$$\Delta G = G_f - G_i < 0$$



single electon transistor SET



$$E = \frac{Q_1^2}{2C_1} + \frac{Q_2^2}{2C_2} + \frac{Q_g^2}{2C_g}$$

$$Q_i = Q_2 + Q_g - Q_1 \qquad Q_i = -ne$$

$$V_{SD} = V_1 + V_2 = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$

$$V_1 = \frac{(C_2 + C_g)V_{SD} - C_g V_g + ne}{C_\Sigma}$$

$$V_2 = \frac{C_1 V_{SD} + C_g V_g - ne}{C_\Sigma} \qquad C_\Sigma = C_1 + C_2 + C_g$$

n_1 number of electrons that tunneled through junction 1

n_2 junction 2

$$n = n_1 - n_2$$

$$G(V_g, V_{SD}, n_1, n_2) = \frac{(ne - C_g V_g)^2}{2 C_\Sigma} - \frac{n_1 C_1 + n_2 (C_2 + C_g)}{C_\Sigma} e V_{SD}$$

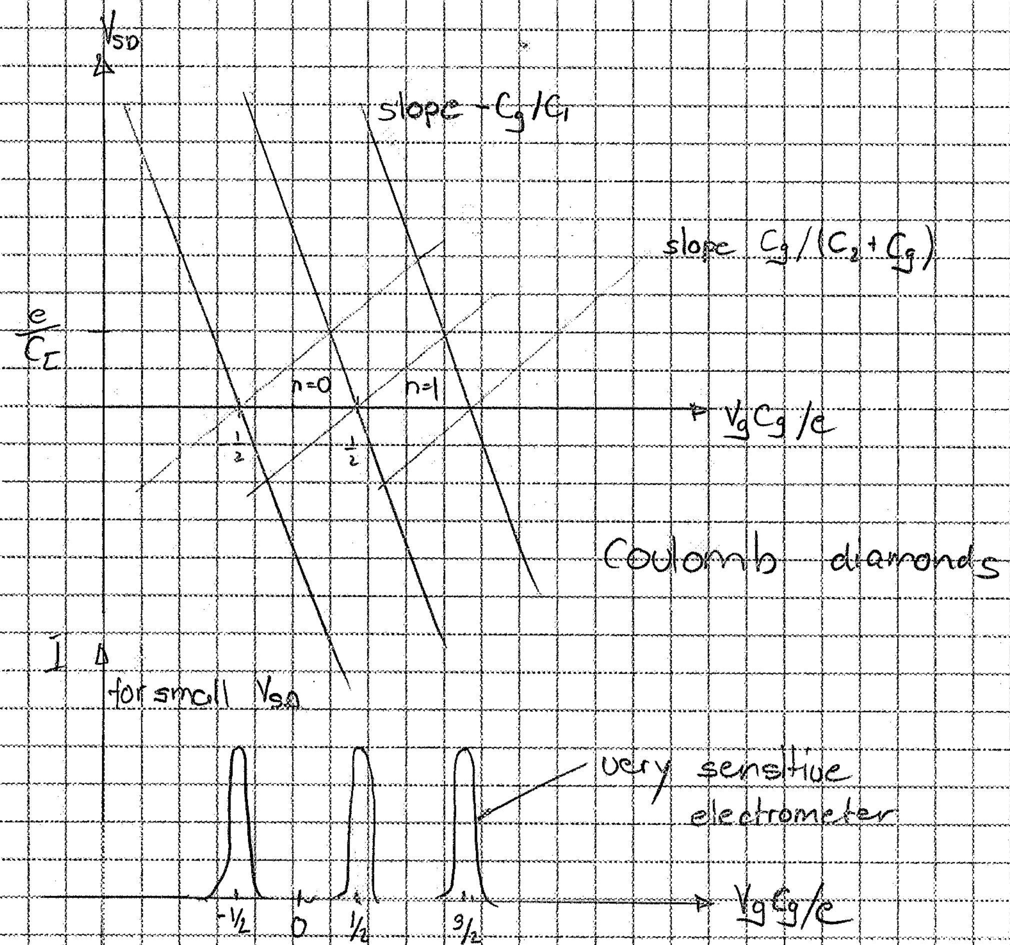
change of free energy due to charge tunneling

$$\Delta G(n_1 + 1, n_2) = \frac{e}{C_\Sigma} \left[\frac{e}{2} - C_1 V_{SD} - C_g V_g + ne \right]$$

$$\Delta G(n_1 + 1, n_2 + 1) = \frac{e}{C_\Sigma} \left[-\frac{e}{2} + C_g V_g - (C_2 + C_g) V_{SD} - ne \right]$$

tunneling if $\Delta G < 0$ (for $T = 0$)

for a current, both ΔG have to be < 0



V_{SD}
slope $-C_g/C_1$
slope $C_g/(C_2+C_g)$
$\frac{e}{C_\Sigma}$
n=0
n=1
$V_g C_g/e$
$\frac{1}{2}$
$\frac{1}{2}$
Coulomb diamonds
I
for small V_{SD}
very sensitive electrometer
$-1/2$
0
$1/2$
$3/2$
$V_g C_g/e$